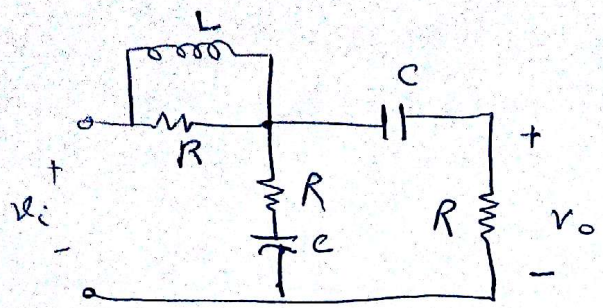
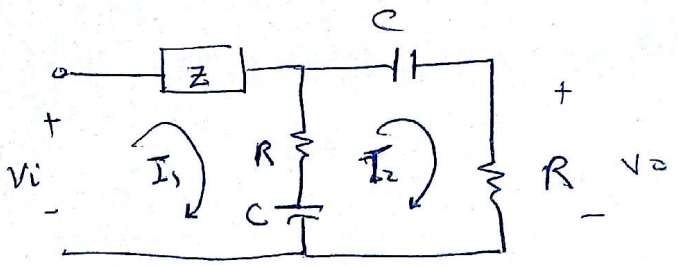
Final exam

Q (1) [a] Find the transfer Functions:



$$Z = R \parallel sL = \frac{sLR}{R + sL}$$



$V_o = I_2(s)\, R$

$$\begin{vmatrix} Z + R + \frac{1}{sC} & -R - \frac{1}{sC} \\ -R - \frac{1}{sC} & 2R + \frac{2}{sC} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} V_i \\ 0 \end{vmatrix}$$

$$I_2 = \frac{\begin{vmatrix} Z + R + \frac{1}{sC} & V_i \\ -R - \frac{1}{sC} & 0 \end{vmatrix}}{\left(Z + R + \frac{1}{sC}\right)\left(2R + \frac{2}{sC}\right) - \left(R + \frac{1}{sC}\right)^2}$$

$$I_2 = \frac{V_i \left(R + \frac{1}{sC}\right)}{2RZ + \frac{2Z}{sC} + 2R^2 + \frac{2R}{sC} + \frac{2R}{sC} + \frac{2}{(sC)^2} - R^2 - \frac{2R}{sC} - \frac{1}{(sC}}$$

$$V_o = I_2 \cdot R = \frac{V_i \left(R + \frac{1}{sC}\right) R}{Z\left(2R + \frac{2}{sC}\right) + R^2 + \frac{2R}{sC} + \frac{1}{(sC)^2}}$$

$$\therefore \frac{V_o}{V_i} = \frac{R^2 + \frac{R}{sC}}{\frac{sLR}{R + sL}\left[2R + \frac{2}{sC}\right] + R^2 + \frac{2R}{sC} + \frac{1}{(sC)^2}}$$

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$$\frac{V_o}{V_i} = \frac{S^2C^2R^2 + SCR}{\frac{S^2CLR}{R+SL}[2RSC+2] + S^2C^2R^2 + 2RSC + 1}$$

$$\frac{V_o}{V_i} = \frac{\left((SCR)^2 + SCR\right)(R+SL)}{S^2CLR[2RSC+2] + S^2C^2R^3 + 2R^2SC + R + S^3LC^2R^2 + 2S^2LCR + SL}$$

$$\frac{V_o}{V_i} = \frac{S^2C^2R^3 + SCR^2 + S^3C^2R^2L + S^2CLR}{2S^3C^2LR^2 + 2S^2CLR + S^2C^2R^3 + 2R^2SC + R + S^3LC^2R^2 + 2S^2LCR + SL}$$

$$\frac{V_o}{V_i} = \frac{S^3C^2R^2L + S^2(C^2R^3 + CLR) + SCR^2}{S^3(2C^2LR^2 + LC^2R^2) + S^2(2CLR + C^2R^3 + 2LCR) + S(2R^2C + L) + R}$$

$$\frac{V_o}{V_i} = \frac{S^3\frac{C^2R^2L}{(2C^2LR^2 + LC^2R^2)} + S^2\left(\frac{C^2R^3 + CLR}{3LC^2R^2}\right) + S\left(\frac{CR^2}{3LC^2R^2}\right)}{S^3 + S^2\left(\frac{3LCR + C^2R^3}{3LC^2R^2}\right) + S\left(\frac{2R^2C + L}{3LC^2R^2}\right) + \frac{R}{3LC^2R^2}}$$

$$\frac{V_o}{V_i} = \frac{S^3\left(\frac{1}{3}\right) + S^2\left(\frac{C^2R^3 + CLR}{3LC^2R^2}\right) + S\left(\frac{CR}{3LC}\right)}{S^3 + S^2\left(\frac{1}{CR} + \frac{R}{3L}\right) + S\left(\frac{2}{3LC} + \frac{1}{3C^2R^2}\right) + \frac{1}{3LC^2R}}$$

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[b]

1) $S^5 + 8S^4 + 2S^3 + 4S^2 + 2S + 4 = 0$

+	S^5	1	2	2
+	S^4	8	4	4
+	S^3	3/2	3/2	
−	S^2	−4	4	
+	S^1	3		
+	S^0	4		

$n = 5$

n_+	n_0	n_-
2	0	3

L system Unstable

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[2] $S^6 + S^5 + 2S^4 + S^3 + 3S^2 + 2S + 2 = 0$

S^6	1	2	3	2
S^5	1	1	2	
S^4	1	1	2	
S^3	4	2		
S^2	1/2	2		
S^1	-14			
S^0	2			

A(s) ↑

$A(s) = S^4 + S^2 + 2$

$4S^3 + 2S$

4 2

$n = 6$

n_+	n_0	n_-
2	0	4

System Unstable

3) $S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0$

S^5	1	2	3
S^4	1	2	5
S^3	2	-2	
S^2	3	5	
S^1	$-\frac{16}{3}$		
S^0	5		

0 ← -2

+2 0

2 -2 shift

$n = 5$

$n_+ = 2 \quad n_0 = 0 \quad n_- = 3$

System Unstable

Q (2)

a) $G = \frac{25}{S(S+4)} \qquad H = \frac{A}{25}$

Open Loop transfer Fun: $GH = \frac{A}{S(S+4)}$

type : 1

order : 2

Closed Loop transfer Function $\frac{25}{S(S+4)+A}$

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ii) $s(s+4)+A=0$

$s^2+4s+A=0$

s^2	1	A	/
s	4		
s^0	A		

Closed Loop Characteristic $1+GH = s(s+4)+A$

$A > 0$

at $A=0$ critically damped.

iii) $K_p = \lim_{s\to 0} GH = \lim_{s\to 0} \frac{A}{s(s+4)} = \infty$

$K_v = \lim_{s\to 0} sGH = \lim_{s\to 0} \frac{A}{s+4} = \frac{A}{4}$

$K_a = \lim_{s\to 0} s^2 GH = \lim_{s\to 0} \frac{As}{s+4} = 0$

iv) Steady state error : $sse = \frac{x}{1+K_p} + \frac{x}{K_v} + \frac{x}{K_a}$

For $r(t) = 2 - 4t + 0\cdot u(t) + 0\frac{t^2}{2}$

$= \frac{0}{1+K_p} + 2 - \frac{4}{A/4} + \frac{0}{0} = \frac{0}{1+\infty} + 2 - \frac{16}{A} + \frac{0}{0} = \infty$

v) $A = 25$

$T.F = \frac{25}{s^2+4s+25} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\omega_n^2 = 25 \qquad \omega_n = 5 \quad rad/sec$

$2\xi\omega_n = 4 \longrightarrow \xi = \frac{4}{2*5} = 0.4$

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.6855 \ sec$

$t_s = \frac{4}{\xi\omega_n} \longrightarrow$ (2% error)

IF (5% error) $\longrightarrow t_s = \frac{3}{\xi\omega_n}$

$t_s = 2 \ sec$

$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.2538$

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$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$$\phi = \cos^{-1}\xi = 66.42^\circ = 1.15 \text{ rad}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4.58$$

$$\xi\omega_n = 2$$

$$y(t) = 1 - 1.09\, e^{-2t} \sin(4.58t + 1.16)$$

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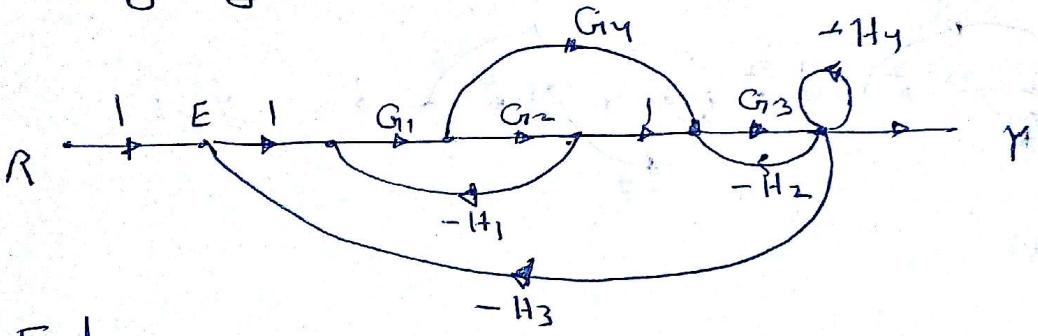
[b] Using signal flow graph



Find

$$\frac{Y(s)}{R(s)} \quad < \quad \frac{Y(s)}{E(s)}$$

$$l_1 = -H_4$$

$$l_2 = -G_1 G_2 H_1$$

$$l_3 = -G_3 H_2$$

$$l_4 = -G_1 G_2 G_3 H_3$$

$$l_5 = -G_1 G_4 G_3 H_3$$

* Non-touching loops:

$$l_1 l_2 = G_1 G_2 H_1 H_4$$

$$l_2 l_3 = G_1 G_2 G_3 H_1 H_2$$

$$\Delta = 1 - \Sigma l_i + \Sigma l_i l_j$$

$$\Delta = 1 + H_4 + G_1 G_2 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + G_1 G_4 G_3 + G_1 G_2 H_1 H_4 + G_1 G_3 H_1 H_2$$

* Forward paths:

$$P_1 = G_1 G_2 G_3$$

$$P_2 = G_1 G_4 G_3$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\frac{Y(s)}{R(s)} = \frac{\Sigma P_i \Delta_i}{\Delta} = \frac{G_1 G_2 G_3 + G_1 G_3 G_4}{\Delta}$$

$$\frac{Y(s)}{E(s)} = \frac{Y(s)}{R(s)} \cdot \frac{1}{E(s)/R(s)} = \frac{G_1 G_2 G_3 + G_1 G_3 G_4}{\Delta} \cdot \frac{\Delta}{\Sigma P_i \Delta_i}$$

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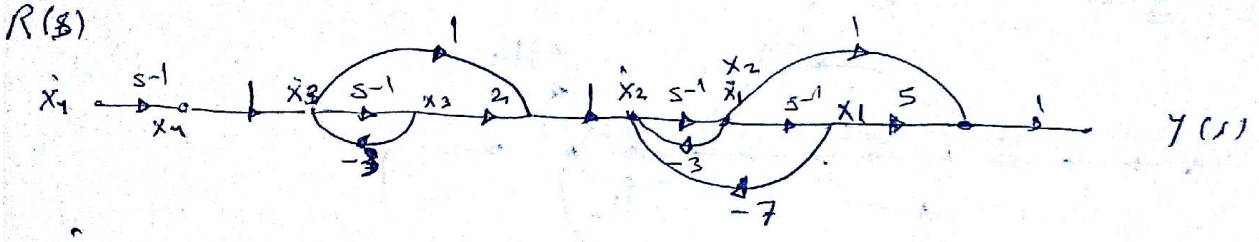
$P_1 = 1$ $\quad \Delta = 1 + G_1G_2H_1 + G_3H_2 + H_4 + G_1G_2G_3H_1H_2 + G_1G_4H_4$ [6]

$$\frac{Y(s)}{E(s)} = \frac{G_1G_2G_3 + G_1G_4G_3}{1 + G_1G_2H_1 + G_3H_2 + H_4 + G_1G_2G_3H_1H_2 + G_1G_2H_1H_4}$$

Q(3) The transfer function

$$G(s) = \frac{(s+2)(s+5)}{s(s+3)(s^2+3s+7)}$$

pole - Zero form: $\frac{1}{s} \cdot \frac{s+2}{s+3} \cdot \frac{s+5}{s^2+3s+7}$



$$\dot{x}_4 = R(t)$$

$$\dot{x}_3 = -3x_3 + x_4$$

$$\dot{x}_2 = \dot{x}_3 + 2x_3 + -3x_2 - 7x_1 = -3x_3 + x_4 + 2x_3 - 3x_2 - 7x_1$$

$$= -7x_1 - 3x_2 - x_3 + x_4$$

$$\dot{x}_1 = x_2 \quad , \quad y(t) = x_2 + 5x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -7 & -3 & -1 & 1 \\ 0 & 0 & -3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} R(t)$$

$$y(t) = \begin{bmatrix} 5 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\, R(t)$$

another representation "controllable"

$$G(s) = \frac{s^2 + 7s + 10}{s^4 + 6s^3 + 16s^2 + 21s}$$

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$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{vmatrix} = \left|\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -21 & -16 & -6 \end{array}\right| \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix} R(t)$$

$$y(t) = \begin{bmatrix} 10 & 7 & 1 & 0 \end{bmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} + [0]\, R(t)$$

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[b]

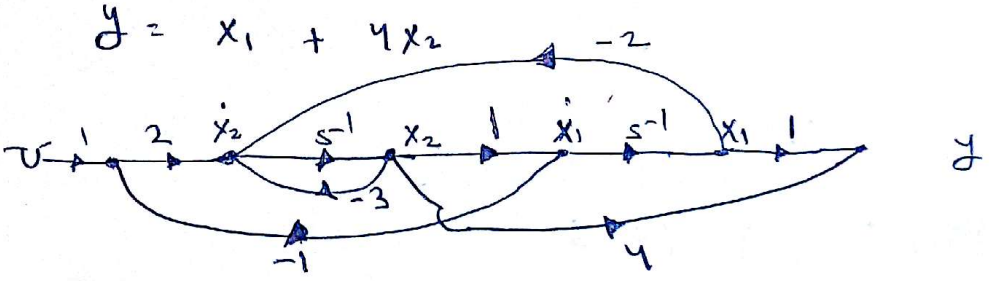
$$\dot{X} = \underset{A}{\begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix}} X + \underset{B}{\begin{vmatrix} -1 \\ 2 \end{vmatrix}} u$$

$$y = \underset{C}{\begin{bmatrix} 1 & 4 \end{bmatrix}} X$$

$$\dot{x}_1 = x_2 - u$$

$$\dot{x}_2 = -2x_1 - 3x_2 + 2u$$

$$y = x_1 + 4x_2$$



* Controllable

$$M_c = \begin{pmatrix} B & AB \end{pmatrix} \qquad B = \begin{vmatrix} -1 \\ 2 \end{vmatrix}$$

$$AB = \begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix} \begin{vmatrix} -1 \\ 2 \end{vmatrix} = \begin{vmatrix} 2 \\ -4 \end{vmatrix} \qquad A = \begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix}$$

$$M_c = \begin{vmatrix} -1 & 2 \\ 2 & -4 \end{vmatrix} = 0$$

∴ uncontrollable ⟵ because $M_c = 0$

$$M_o = \begin{pmatrix} C \\ CA \end{pmatrix} \qquad C = \begin{vmatrix} 1 & 4 \end{vmatrix} \qquad A \begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix}$$

$$CA = \begin{vmatrix} 1 & 4 \end{vmatrix} \begin{vmatrix} 0 & 1 \\ -2 & -3 \end{vmatrix} = \begin{vmatrix} -8 & -11 \end{vmatrix}$$

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$$M_o = \begin{vmatrix} 1 & 4 \\ 8 & -11 \end{vmatrix} \neq 0$$

∴ System is observable.

3) Find transfer Function by signal Flow graph

$L_1 = \frac{-3}{s}$ $\quad P_1 = \frac{2}{s^2}$ $\quad \Delta_1 = 1$

~~$L_2 = \frac{-2}{s}$~~ $\quad P_2 = \frac{-1}{s}$ $\quad \Delta_2 = 1 + \frac{3}{s}$

$L_2 = \frac{-2}{s^2}$ $\quad P_3 = \frac{8}{s}$ $\quad \Delta = 1$

$$T.F = \frac{\sum P_k \Delta_k}{\Delta} = \frac{7s+7}{s^2+3s+2}$$

Q [4]

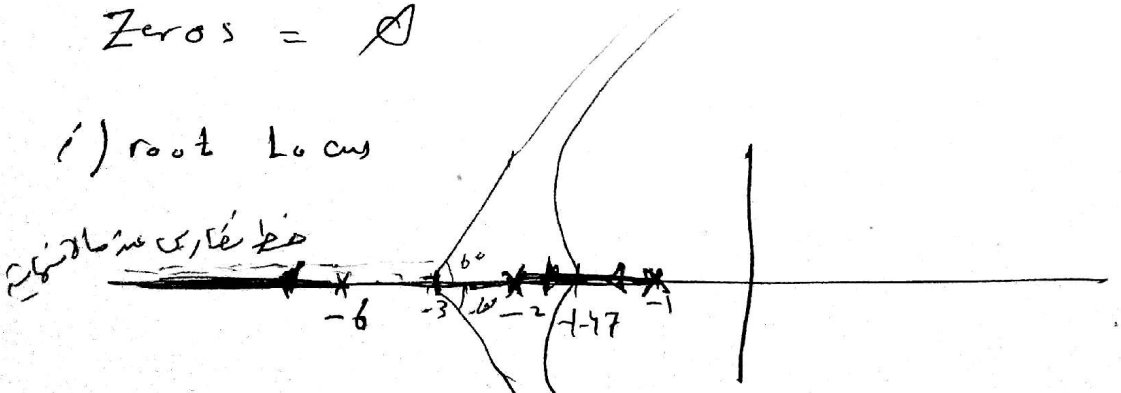
$$G\,H = \frac{K}{(s+1)(s+2)(s+6)}$$

[1]

poles = $[-1, -2, -6]$ $\quad np = 3$

Zeros = $\emptyset$

i) root Locus



$$\delta_c = \frac{\sum P - \sum z}{np - nz} = \frac{(-1)+(-2)+(-6)}{3} = -3$$

$n_o = np - nz = 3$

$\theta_1 = 60, \quad 180, \quad -60$

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$$GH = 1 + \frac{K}{(s+1)(s+2)(s+6)} = 0$$

$$|K| = s^3 + 9s^2 + 20s + 12$$

$$\frac{dK}{ds} = 3s^2 + 18s + 20 = 0$$

$s_1 = -1.472$ ∠ $s_2 = -4.52$ → مرفوض خارج منطقة Root Locus

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② range of K

$$s^3 + 9s^2 + 20s + 12 + K = 0$$

s^3	1	20
s^2	9	$12+K$
s^1	$\frac{180-(12+K)}{9}$	
s^0	$12+K$	

$$\frac{180-(12+K)}{9} > 0$$

$12 + K < 180$

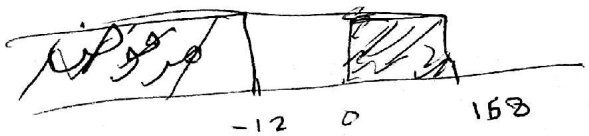
$K < 168$

$12 + K > 0$

$K > -12$

$K > 0$

$0 < K < 168$ ∠



$[0, 168]$

$\xi = 1$ $\cos^{-1} 1 = 0$

③ critically

From sketch

$$K = \frac{\prod l_p}{\prod l_z} = \frac{l_{p_1} l_{p_2} l_{p_3}}{1} = \frac{0.47 \times 0.53 \times 4.53}{1}$$

Zero يوجد ← 1

$$= 1.128$$

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